Impossibility Results for Indifferentiability with Resets

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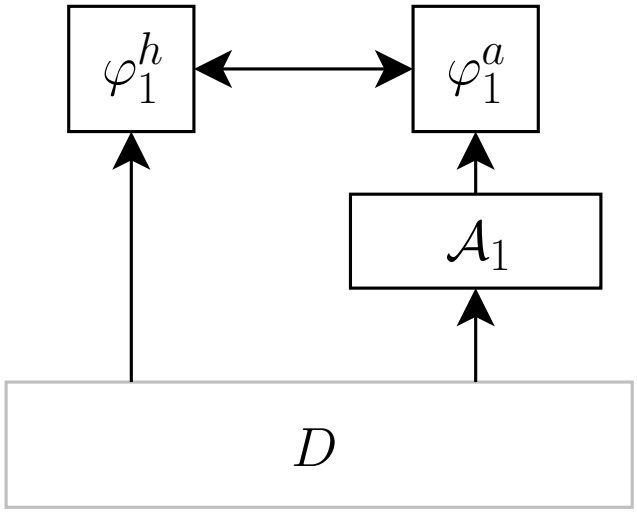
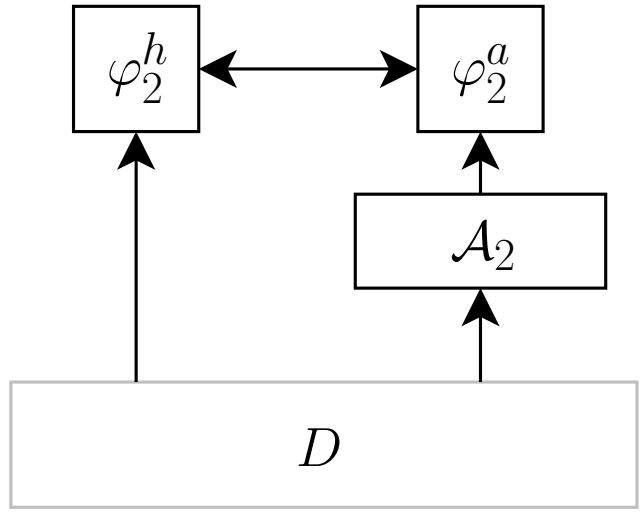
Indifferentiability





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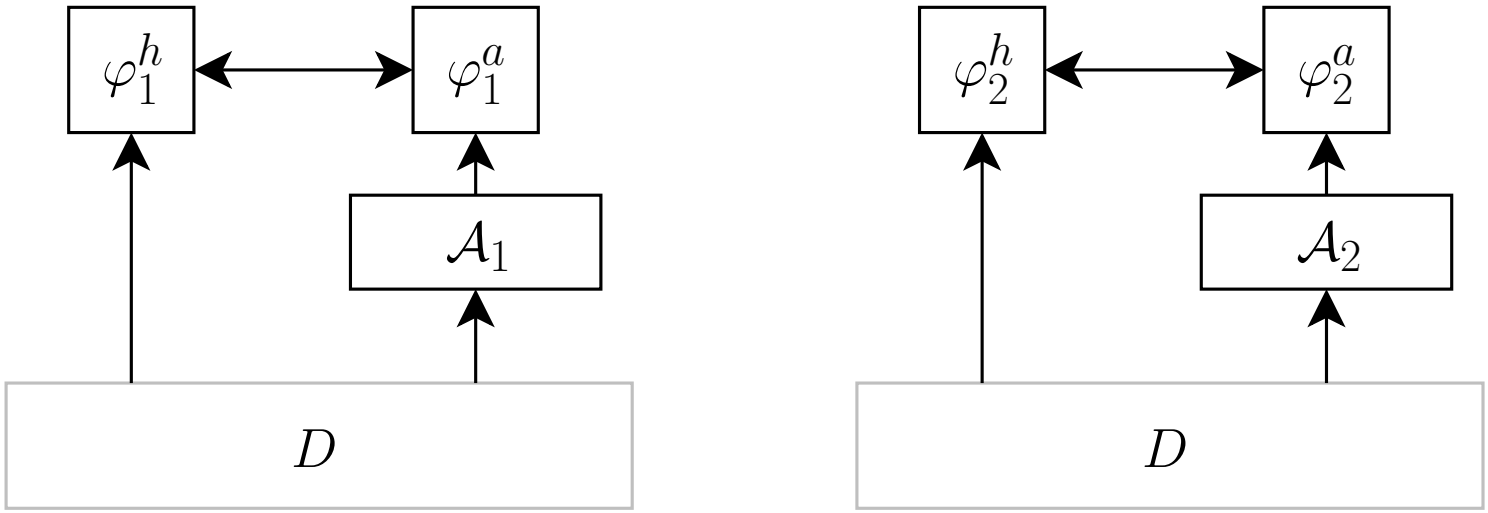

Definition

Functionality φ_1 is at least as secure as φ_2 if for any distinguisher D and adversary $\mathcal{A}_1$ there exist adversary $\mathcal{A}_2$ such that

$$\left|\mathbf{Pr}\left(D_{\varphi_1}^{\mathcal{A}_1} \to 1\right) - \mathbf{Pr}\left(D_{\varphi_2}^{\mathcal{A}_2} \to 1\right)\right| \text{ is small}$$

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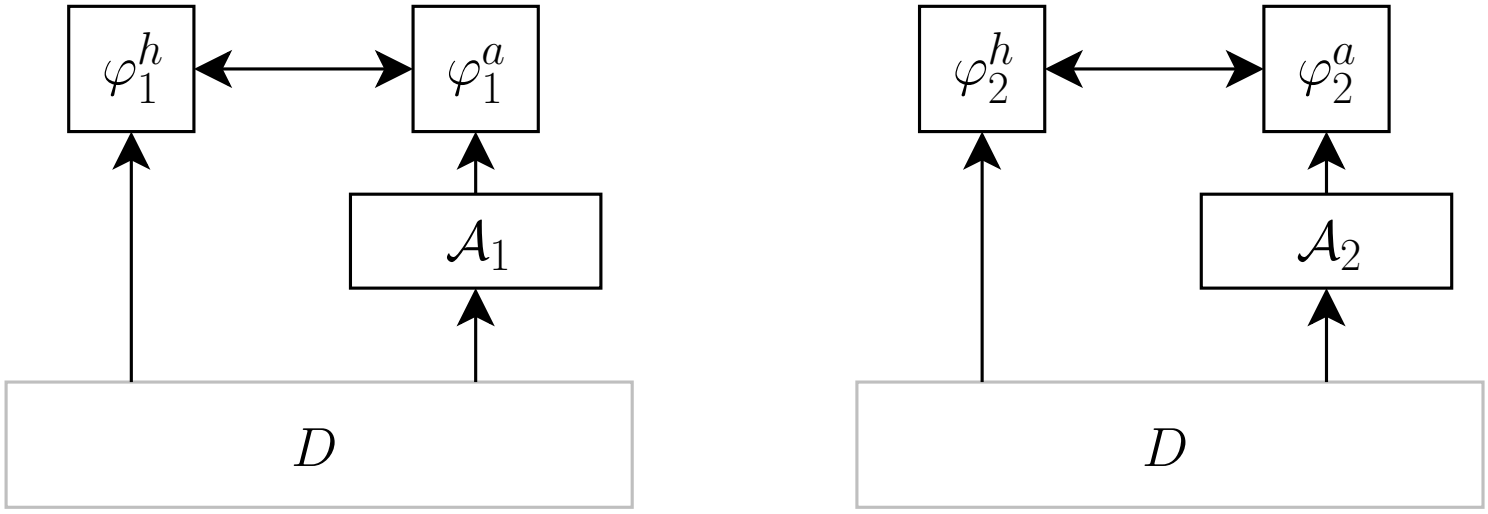

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- Original framework of Maurer et al. (MRH, TCC'04)
- Popularized for domain extenders by Coron et al. (CDMP, Crypto'05)

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"hash function can replace RO in any RO-secure scheme"

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Server	**Client**
(knows: m)	(knows: m)

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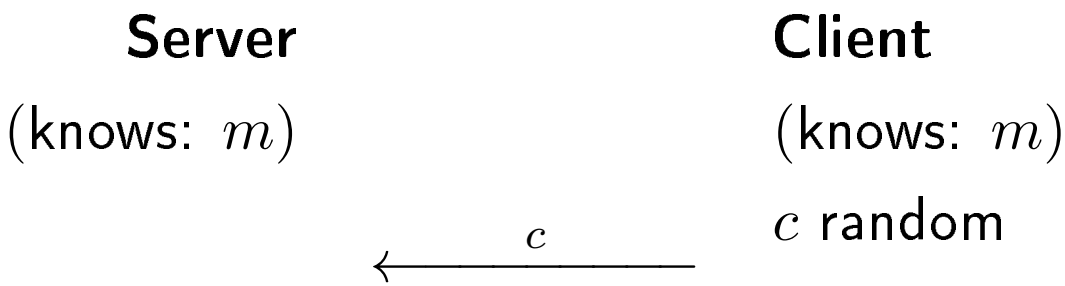

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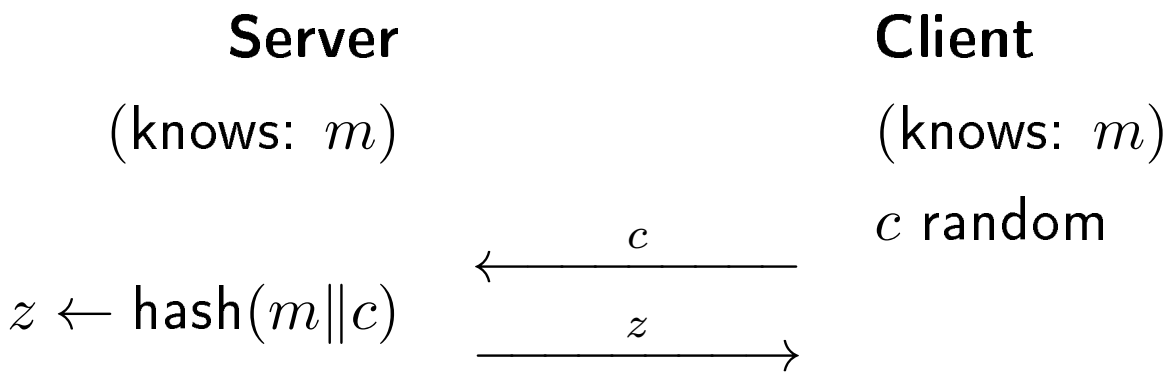

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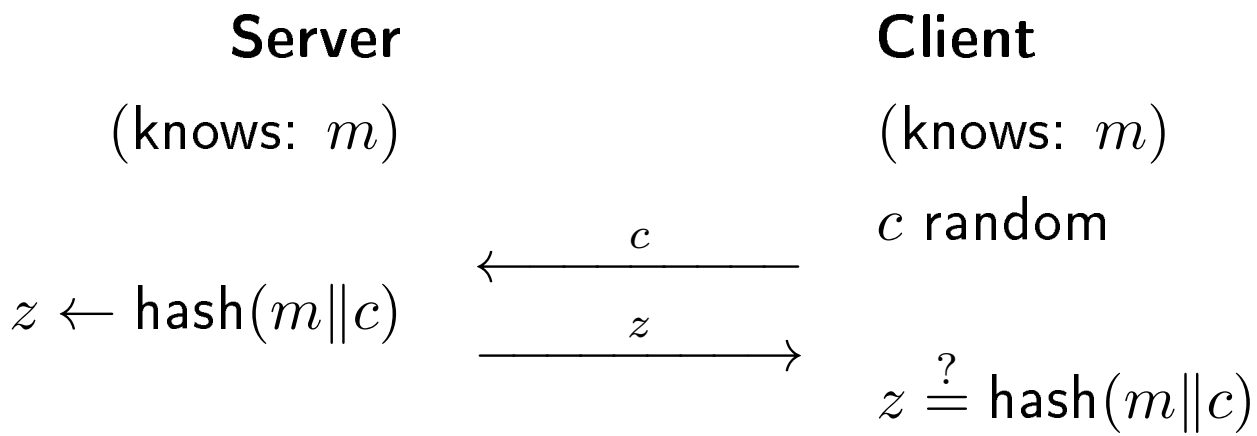

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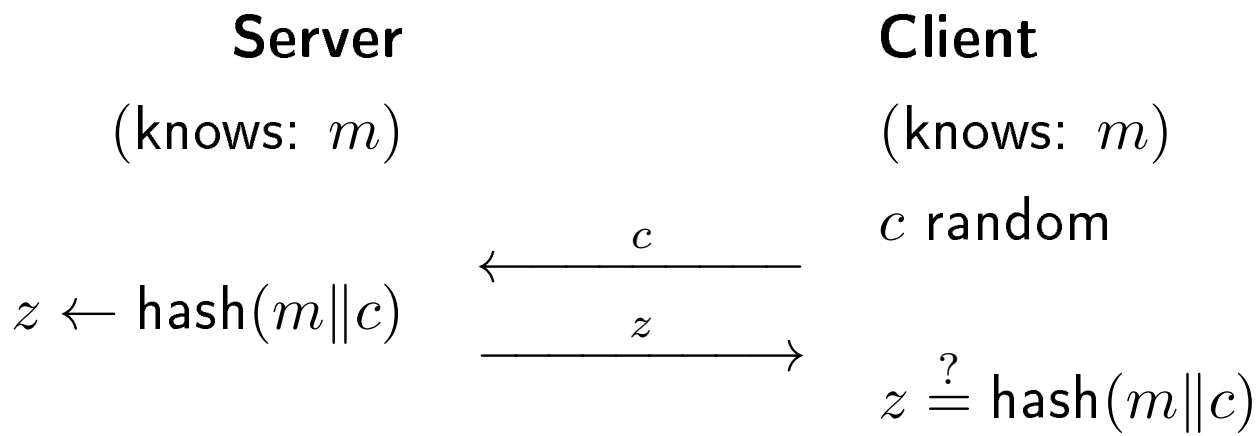

- Secure in ROM, but insecure for some indifferentiable hash functions

Remarks

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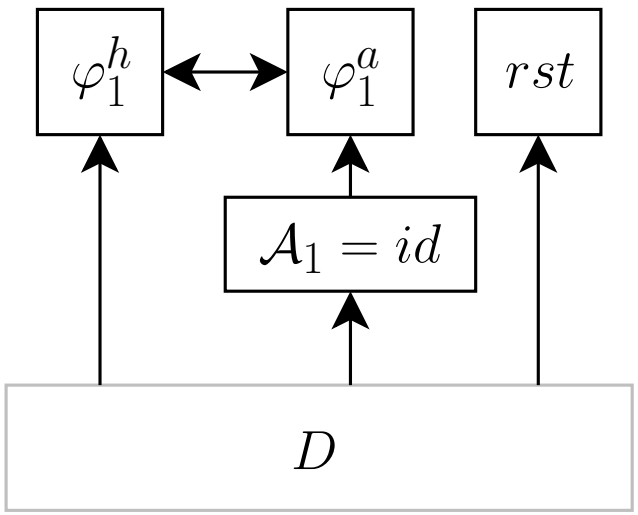
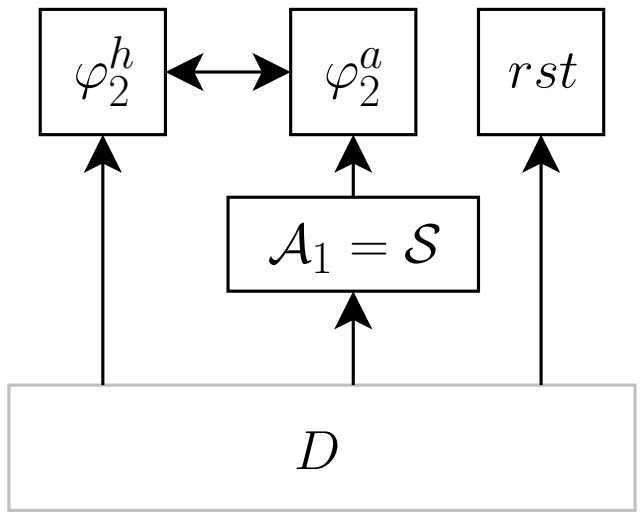
- Indifferentiability is general enough due to MRH's random systems
- Issue comes from CDMP's definition that does not allow distinguishers to manipulate states
- All indifferentiability proofs following CDMP's model limit types of distinguishers (their power) that are covered

Reset-Indifferentiability

- RSS introduce reset-indifferentiability
- Additional procedure rst that when called initializes all of $\mathcal{A}$'s variables





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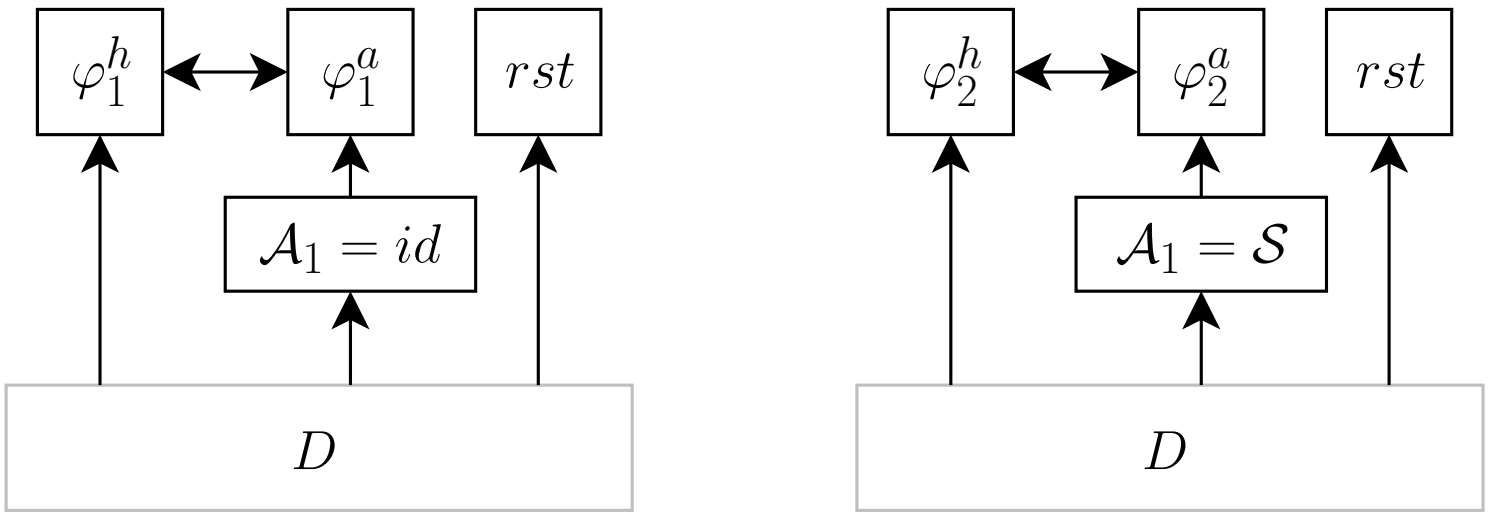
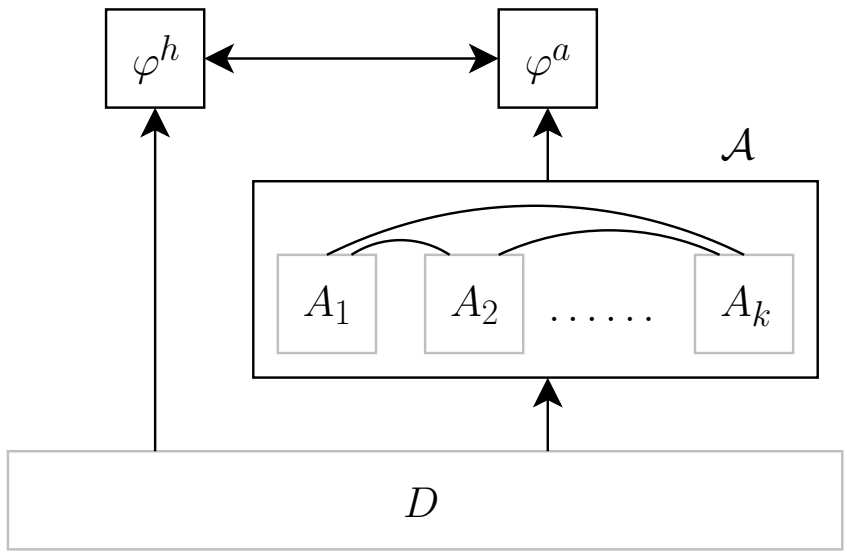


- RSS: impossibility of reset-indifferentiable one-pass domain extenders
  - Applies to MD, chopMD, pfMD, EMD, Sponge, ...
- Model is too strong

Formalizing Reset-Indifferentiability

- Our approach: explicitly model communication between adversaries
  - Storage procedures for any pair of adversaries
  - Reset corresponds to change of adversary



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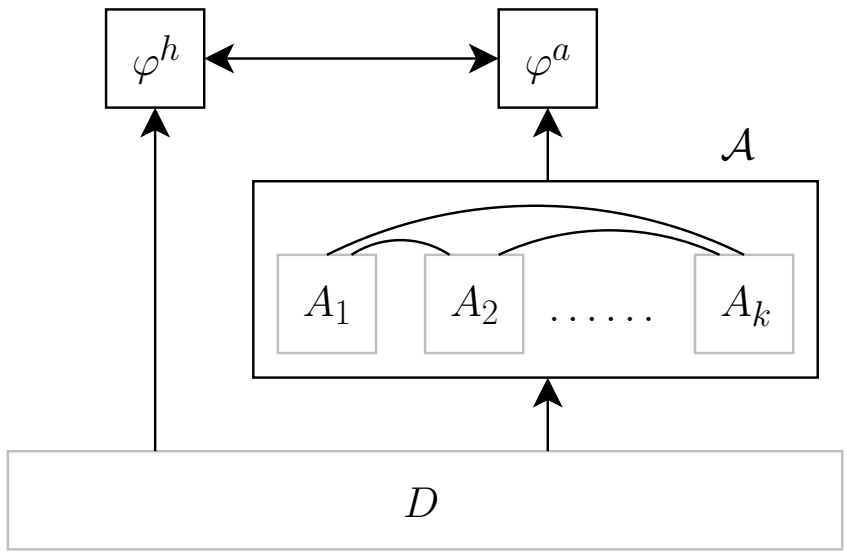

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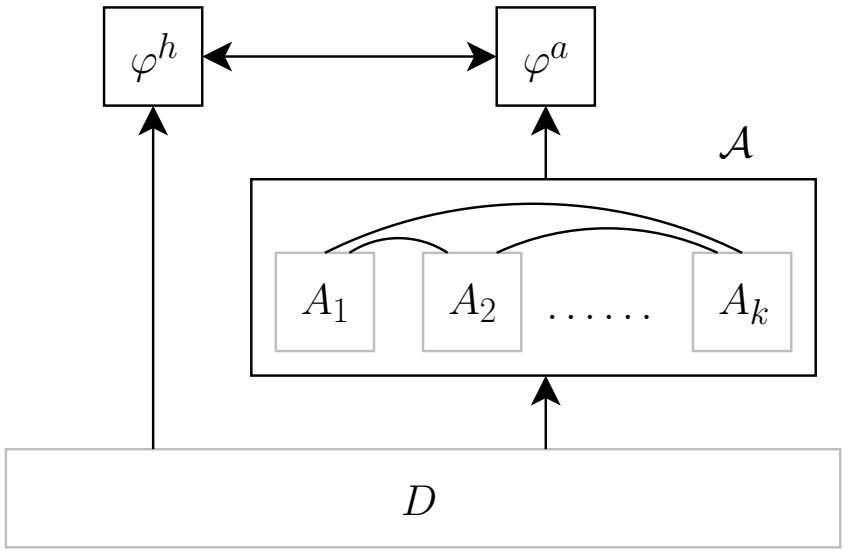

- Model allows for whole spectrum of indifferentiability notions
- k-stage distinguishers
  - 1-stage: regular indifferentiability
  - CRP is 2-stage

$\mathcal{G}$-Indifferentiability

- Let $\mathcal{G}$ be a class of distinguishers
- E.g.: 1-stage, 1-stage $+$ CRP, 2-stage, $\ldots$
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Theorem

There exists n-stage distinguisher D such that $\forall\ \mathcal{A}$:

$$\left|\mathbf{Pr}\left(D_{(F,\pi)} \to 1\right) - \mathbf{Pr}\left(D^{\mathcal{A}}_{RO} \to 1\right)\right| \geq 1 - \left(\frac{q}{2^{M-a}} + \frac{1}{2^H}\right)$$

where $\mathcal{A}$ makes q RO-queries, D makes one message evaluation

- Can be made arbitrarily close to $1 - 1/2^H$

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Proof Idea

- Distinguisher evaluates oracles for some $m \in \{0,1\}^M$
- Resets before every $\mathcal{A}$-call
- At last $\mathcal{A}$-call: learns at most $a \ll M$ bits of m

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Proof Idea

- Distinguisher D is a combination of two distinguishers P and Q
  - If F is "roughly" one-pass: previous distinguisher (call it P) works
  - Otherwise, new distinguisher Q: manipulates m in a sophisticated way
- Distinguisher D chooses m runs either P or Q (depending on m)

Comparison with Resource-Restricted Indifferentiability

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- DGHM's work:
  - Focus on reductions between "resources"
  - Target impossibility results
- Our work:
  - Focus on indifferentiability as way to guarantee security against a variety of attacks
  - More suitable to discover possibilities for composition
  - It shows that there are many different notions to consider

Conclusions

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  $\longrightarrow$ More natural classes $\mathcal{G}$?
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Thank you for your attention!