μKummer: efficient hyperelliptic signatures and key exchange on microcontrollers

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ESC, 20th January 2017

Outline

- Introduction
- High level signature and key exchange schemes
- Jacobian and Kummer arithmetic
- Implementation details
- Results and comparison

Summary of contributions

1. First software-only implementation of hyperelliptic-curve cryptography on microcontrollers (AVR ATmega and ARM Cortex M0)
2. First implementation of a signature scheme based on a Kummer surface
3. Significant improvement over state-of-the-art in terms of speed, size and stack usage

Software in the public domain. Available at

`http://www.cs.ru.nl/~jrenes/`

Elliptic curve cryptography

- Elliptic curve E defined by an equation

$$y^2 = f(x)$$

  where $\deg(f) = 3, 4$
- The points on E form a group, with corresponding operations

$$P \mapsto [2]P, \quad (P, Q) \mapsto P + Q$$

- Two main use-cases:
  - Key exchange: relies on scalar multiplication $(k, P) \to [k]P$
  - Signatures: relies on scalar multiplication and addition

x-only scalar multiplication

- There exists a map $E \to E/\{\pm 1\}$ such that $(x, y) \mapsto x$
- The points on $E/\{\pm 1\}$ do not form a group, but there are operations

$$\begin{aligned} \texttt{xDBL}: &\quad x_P \mapsto x_{2P} \\ \texttt{xADD}: &\quad (x_P, x_Q, x_{P\pm Q}) \mapsto x_{P\pm Q} \end{aligned}$$

- Efficient x-only scalar multiplication on $E/\{\pm 1\}$, based on the Montgomery ladder (e.g. Curve25519 [Ber06])
- Main use-case: key exchange
- No group law on $E/\{\pm 1\}$, hence cannot do standard signatures directly (e.g. Ed25519 [Dul+15])

Moving to genus 2

- Hyperelliptic curve $\mathcal{E}$ defined by an equation

$$y^2 = f(x)$$

  where $\deg(f) = 5, 6$
- The Jacobian $\mathcal{J}$ forms a group, with corresponding operations

$$P \mapsto [2]P, \quad (P, Q) \mapsto P + Q$$

- Two main use-cases:
  - Key exchange: relies on scalar multiplication $(k, P) \to [k]P$
  - Signatures: relies on scalar multiplication and addition
- Note that operations on $\mathcal{J}$ are slow and not constant-time!

- There exists a map $\mathcal{J} \to \mathcal{J}/\{\pm 1\}$. We call

$$\mathcal{K} := \mathcal{J}/\{\pm 1\}$$

  the Kummer surface.
- The points on $\mathcal{K}$ do not form a group, but there are operations

$$\texttt{xDBL}: \quad x_P \mapsto x_{2P}$$
$$\texttt{xADD}: \quad x_P, x_Q, x_{P \pm Q} \mapsto x_{P \pm Q}$$

- Efficient scalar multiplication on $\mathcal{K}$, based on the Montgomery ladder
- Main use-case: key exchange
- No group law on $\mathcal{K}$, hence cannot do standard signatures

Curve-based cryptography

Genus	$g = 1$	$g = 2$
Curve	Elliptic curve E	Hyperelliptic curve $\mathcal{E}$
Cryptographic group	Points	Jacobian
Kummer	$E/\{\pm 1\}$	$\mathcal{K} := \mathcal{J}/\{\pm 1\}$

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- Operations

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$$\texttt{ADD}: \quad P, Q \mapsto P + Q$$

- Two main use cases:
  - Key exchange: relies on scalar multiplication $k, P \to [k]P$
  - Signatures: relies on scalar multiplication and addition
- Operations on $\mathcal{J}$ are hard to make fast and constant-time!

Curve-based cryptography

Genus	$g = 1$	$g = 2$
Curve	Elliptic curve E	Hyperelliptic curve $\mathcal{E}$
Cryptographic group	Points	Jacobian
Kummer	$E/\{\pm 1\}$	$\mathcal{K} := \mathcal{J}/\{\pm 1\}$

- Corresponds to $(x, y) \mapsto x$
- Not a group. Use x-only operations

$$\mathtt{xDBL}: \quad x_P \mapsto x_{[2]P}$$
$$\mathtt{xADD}: \quad x_P, x_Q, x_{P\pm Q} \mapsto x_{P\mp Q}$$

- Scalar multiplication via the Montgomery ladder (e.g. Curve25519 [Ber06])
- Main use case: key exchange
- No signatures (e.g. Ed25519 [Ber+12])

Curve-based cryptography

Genus	$g = 1$	$g = 2$
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- Scalar multiplication via the Montgomery ladder
- Main use case: key exchange
- No signatures (need Jacobian)

(Hyper)elliptic curve crypto summarized

The situation in short:

- $E \leftrightarrow \mathcal{J}$
  - Key exchange ✔
  - Signatures ✔
- $E/\{\pm 1\} \leftrightarrow \mathcal{K}$
  - Key exchange ✔
  - Signatures ✗

(Hyper)elliptic curve crypto summarized

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  - Signatures ✔
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  - Key exchange ✔
  - Signatures ✗

New result [CCS16]; use $\mathcal{K}$ to do fast signatures on $\mathcal{J}$:

$$\begin{array}{ccc} \mathcal{J} & \dashrightarrow & \mathcal{J} \\ \downarrow & & \uparrow \\ \mathcal{K} & \longrightarrow & \mathcal{K} \end{array} \qquad \begin{array}{ccc} P & \dashrightarrow & [k]P \\ \downarrow & & \uparrow \\ x_P & \longrightarrow & (x_{[k]P}, x_{[k+1]P}, x_P) \end{array}$$

PpR: "`Project`-pseudomultiply-`Recover`"

Implementation results

- On larger platforms speed records are challenged by Kummer surface implementations [CL15; Ber+14]
- Speed records for 128-bit secure key exchange and signatures on microcontrollers held by elliptic-curve-based schemes

Two interesting questions:

- Q: How well do Kummer-based key exchange schemes perform on microcontrollers?
  - *A: Probably well, but never implemented*
- Q: How do Kummer-based signatures schemes perform?
  - *A: Not clear*

The signature scheme

- Public generator $P \in \mathcal{J}$, 512-bit hash function H, 256-bit secret key d, message M
- Three main functions
  - `keygen`:
    1. $(d'||d'') \leftarrow H(d)$
    2. $Q \leftarrow [16d']P$
  - `sign`:
    1. $(d'||d'') \leftarrow H(d)$
    2. $r \leftarrow H(d''||M)$
    3. $R \leftarrow [r]P$
    4. $h \leftarrow H(R||Q||M)$
    5. $s \leftarrow r - 16h_{128}d' \pmod{\#\mathcal{J}/16}$
    6. $\sigma \leftarrow (h_{128}||s)$
  - `verify`:
    1. $T \leftarrow [s]P + [h_{128}]Q$
    2. $g \leftarrow H(T||Q||M)$
    3. $g_{128} \stackrel{?}{=} h_{128}$

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    2. $Q \leftarrow [16d']P$ (!) Elements of $\mathcal{J}$
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    2. $r \leftarrow H(d''||M)$
    3. $R \leftarrow [r]P$ (!) Elements of $\mathcal{J}$
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  - `sign`:
    1. $(d'||d'') \leftarrow H(d)$
    2. $r \leftarrow H(d''||M)$
    3. $R \leftarrow [r]P$ (!) Scalarmult through $\mathcal{K}$ via PpR
    4. $h \leftarrow H(R||Q||M)$
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    2. $r \leftarrow H(d''||M)$
    3. $R \leftarrow [r]P$
    4. $h \leftarrow H(R||Q||M)$
    5. $s \leftarrow r - 16h_{128}d' \pmod{\#\mathcal{J}/16}$
    6. $\sigma \leftarrow (h_{128}||s)$ (!) Compressed to 384 bits by sending h_{128}
  - `verify`:
    1. $T \leftarrow [s]P + [h_{128}]Q$ (!) Half-size scalar multiplication
    2. $g \leftarrow H(T||Q||M)$
    3. $g_{128} \stackrel{?}{=} h_{128}$

The signature scheme

- Public generator $P \in \mathcal{J}$, 512-bit hash function H, 256-bit secret key d, message M
- Three main functions
  - `keygen`:
    1. $(d' || d'') \leftarrow H(d)$
    2. $Q \leftarrow [16d']P$ (!) Compression of Q
  - `sign`:
    1. $(d' || d'') \leftarrow H(d)$
    2. $r \leftarrow H(d'' || M)$
    3. $R \leftarrow [r]P$ (!) Compression of R
    4. $h \leftarrow H(R || Q || M)$
    5. $s \leftarrow r - 16 h_{128} d' \pmod{\#\mathcal{J}/16}$
    6. $\sigma \leftarrow (h_{128} || s)$
  - `verify`:
    1. $T \leftarrow [s]P + [h_{128}]Q$ (!) Compression of T
    2. $g \leftarrow H(T || Q || M)$
    3. $g_{128} \stackrel{?}{=} h_{128}$

The key exchange scheme

- Public generator $P \in \mathcal{K}$, 256-bit secret key d
- One main function
  - `dh_exchange`:
    1. $Q \leftarrow [d]P$

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The key exchange scheme

- Public generator $P \in \mathcal{K}$, 256-bit secret key d
- One main function
  - `dh_exchange`:
    1. $Q \leftarrow [d]P$ (!) Both `keygen` and `exchange`

- Finite field $\mathbb{F}_q$ with $q = 2^{127} - 1$
- The Gaudry-Schost curve $\mathcal{C}$ is a genus 2 hyperelliptic curve

$$\mathcal{C} : Y^2 = X(X-1)(X-\lambda)(X-\mu)(X-\nu),$$

for constants $\lambda, \mu, \nu \in \mathbb{F}_q$

- Jacobian $\mathcal{J}_{\mathcal{C}}(\mathbb{F}_q)$
- Kummer surface $\mathcal{K}_{\mathcal{C}}(\mathbb{F}_q) := \mathcal{J}_{\mathcal{C}}(\mathbb{F}_q)/\{\pm 1\}$

Function	Domain & Range	**M**	**S**	$\mathbf{m_c}$	**a**	**s**	**I**
`ADD`	$\mathcal{J}_{\mathcal{C}} \to \mathcal{J}_{\mathcal{C}}$	28	2	0	11	24	0
`Project`	$\mathcal{J}_{\mathcal{C}} \to \mathcal{K}_{\mathcal{C}}$	8	1	4	7	8	0
`xDBLADD`	$\mathbb{Z} \times \mathcal{K}_{\mathcal{C}} \to \mathcal{K}_{\mathcal{C}}^2$	7	12	12	16	16	0
`Recover`	$\mathcal{J}_{\mathcal{C}} \times \mathcal{K}_{\mathcal{C}}^3 \to \mathcal{J}_{\mathcal{C}}$	77	8	0	19	10	1

ARM Cortex M0

- 32-bit microcontroller
- Represent elements of $\mathbb{F}_{2^{127}-1}$ with 4 32-bit words (1 bit left)
- 128×128-bit multiplication (`bigint_mul`) and squaring (`bigint_sqr`) from [Dul+15]
  - 2-level Karatsuba multiplication and 2-level Karatsuba squaring
- Reduction (`bigint_red`) based on $2^{128} \equiv 2 \pmod{2^{127}-1}$
- Combined into field multiplication (`gfe_mul`) and squaring (`gfe_sqr`)
- Fast 16×128-bit multiplication by constant (`gfe_mulconst`)
- Inversion (`gfe_invert`) based on $g^{-1} = g^{2^{127}-3}$

Results and comparison

AVR ATmega (scalarmult)

	Imp.	Object	Cycles	Code size	Stack
DH	[LWG14]	256-bit curve	$\approx$ 21 078 200	14 700 bytes	556 bytes
S,DH	[WUW13]	NIST P-256	$\approx$ 34 930 000	16 112 bytes	590 bytes
DH	[HS13]	Curve25519	22 791 579	n/a	677 bytes
DH	[Dul+15]	Curve25519	13 900 397	17 710 bytes	494 bytes
DH	**This work**	$\mathcal{K}_{\mathcal{C}}$	9 513 536	$\approx$ 9 490 bytes	99 bytes
S	**This work**	$\mathcal{J}_{\mathcal{C}}$	9 968 127	$\approx$ 16 516 bytes	735 bytes

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Key exchange: Reducing number of clock cycles by 32%, almost halving code size and reducing stack usage by about 80%

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Signatures: Reducing number of clock cycles by 71%, increasing stack usage by 25%

AVR ATmega (full signatures)

Imp.	Object	Function	Cycles	Stack
[NLD15]	Ed25519	sig. gen.	19 047 706	1 473 bytes
[NLD15]	Ed25519	sig. ver.	30 776 942	1 226 bytes
This work	$\mathcal{J}_{\mathcal{C}}$	`sign`	10 404 033	926 bytes
This work	$\mathcal{J}_{\mathcal{C}}$	`verify`	16 240 510	992 bytes

Almost half the number of cycles, decrease stack usage (code size not reported)

ARM Cortex M0 (scalarmult)

	Imp.	Object	Clock cycles	Code size	Stack
S,**DH**	[WUW13]	NIST P-256	$\approx$ 10 730 000	7 168 bytes	540 bytes
DH	[Dul+15]	Curve25519	3 589 850	7 900 bytes	548 bytes
DH	**This work**	$\mathcal{K}_{\mathcal{C}}$	2 633 662	$\approx$ 4 328 bytes	248 bytes
S	**This work**	$\mathcal{J}_{\mathcal{C}}$	2 709 401	$\approx$ 9 874 bytes	968 bytes

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Key exchange: Reducing number of clock cycles by 27%, halving code size and stack usage

ARM Cortex M0

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S,**DH**	[WUW13]	NIST P-256	$\approx$ 10 730 000	7 168 bytes	540 bytes
DH	[Dul+15]	Curve25519	3 589 850	7 900 bytes	548 bytes
DH	**This work**	$\mathcal{K}_{\mathcal{C}}$	2 633 662	$\approx$ 4 328 bytes	248 bytes
S	**This work**	$\mathcal{J}_{\mathcal{C}}$	2 709 401	$\approx$ 9 874 bytes	968 bytes

Signatures: Reducing number of clock cycles by 75%, increase in code size and stack usage

Thanks for your attention!

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